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# VIRTUAL REPRESENTATION OF IID OBSERVATIONS IN BAYESIAN BELIEF NETWORKS

Robert J. Mislevy



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Local computation for updating Bayesian belief networks proceeds in the context of a "join tree," consisting of subsets of interrelated variables (cliques) joined by their intersection sets in a singly-connected graphical structure. When multiple independent and identically-distributed (IID) observations of a variable can be made, identically structured cliques corresponding to each potential observation appear as terminal nodes in the join tree. This note shows how it is possible to absorb information from an indefinite number of observations of this type without preconstructing and manipulating cliques for all potential observations. An "update & replace" strategy carries the necessary information with only two nodes for a family of IID observations of a variable at any point in time.

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# Virtual Representation of IID Observations in Bayesian Belief Networks

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# Virtual Representation of IID Observations in Bayesian Belief Networks


Abstract

Local computation for updating Bayesian belief networks proceeds in the context of a "join tree," consisting of subsets of interrelated variables (cliques) joined by their intersection sets in a singly-connected graphical structure. When multiple independent and identically-distributed (IID) observations of a variable can be made, identically structured cliques corresponding to each potential observation appear as terminal nodes in the join tree. This note shows how it is possible to absorb information from an indefinite number of observations of this type without preconstructing and manipulating cliques for all potential observations. An "update & replace" strategy carries the necessary information with only two nodes for a family of IID observations of a variable at any point in time.



Key words: Bayesian inference networks, causal probability networks, expert systems, influence diagrams, intelligent tutoring systems, local computation.

# Introduction

Bayesian inference networks (also referred to as causal probability networks and influence diagrams) are systems for representing, and carrying out probability-based inference in, assemblages of interrelated variables (Andreassen, Jensen, & Olesen, 1990; Lauritzen & Spiegelhalter, 1988; Pearl, 1988; Shafer & Shenoy, 1988). Starting from a directed acyclic graphical (DAG) representation of the conditional independence relationships among the variables, a representation of the joint probability distribution is constructed in terms of products of the distributions of subsets of interrelated variables (cliques), divided by distributions of intersecting subsets (clique intersections). A corresponding "join tree" representation of cliques with the property of single-connectness enables coherent propagation of the consequences of new information throughout the network, employing only calculations local to cliques and their immediate neighbors. Commerically available computer programs for structuring and using Bayesian inference include ERGO (Noetic Systems, Inc., 1991) and HUGIN (Andersen, Jensen, Olesen, & Jensen, 1989).

This note presents a technique for reducing the effective computing size of Bayesian inference networks with nodes that represent repeated independent and identically-distributed (IID) observations. It is assumed that all such variables in the class have exactly the same parents and the same conditional probabilities, given values of their parents. Examples are as follows:

- In target shooting, repeated shots at the same target are made under the same conditions. The IID observed variables are individual shot scores, and the shooter's level of skill is the parent. Successive observed shot scores are of interest only insofar as they cumulatively provide increasing precision about the shooter's level of skill.

- In medical diagnosis, unreliable tests may be repeated to increase accuracy. Assuming the "true value" remains constant, the repeated test results are IID, with conditional probability distributions determined by the true physiological state and the common distribution of the measurement errors.

- In educational testing, item responses may be sequentially obtained to randomly-sampled items from a domain. The parent, and the variable of ultimate interest, is a student's probability of correctly answering a randomly-selected item; that is, the

student's "true score." Item responses are modeled as conditionally independent Bernoulli variables, given true score. An extension of this example is used below to exemplify the technique.

- In an intelligent tutoring system (ITS), such as the HYDRIVE system for aircraft hydraulics troubleshooting (Gitomer, Steinberg, & Mislevy, in press), students working through a problem arrive at unique situations in unique ways, depending on their particular sequence of actions. Their actions in any situation are postulated to depend on their values of unobservable variables in a "student model," characterizing key aspects of their system and strategic knowledge. It is not feasible to model every conceivable scenario a priori as a potential observable variable. It may be possible, however, to partition this large observation space into equivalence classes defined in terms of more abstract descriptions of potential outcomes and observations, and to specify corresponding conditional probabilities given values of student-model variables. All parallel observations in such an equivalence class may then be modeled approximately as IID. It will not be known *a priori* how many observations in each equivalence class will be realized for any given student in any given problem.

Our objective is to carry out inference in such situations without explicitly building an inference network for all the IID variables in a class. This can be accomplished if one can structure the join tree so the nodes for variables in a given IID class appear only in terminal nodes, which is straightforward if each such variable is a terminal node in the corresponding influence diagram; i.e., it has no children. This is the case in the settings in which we are interested.

## A Solution

The technique outlined below requires maintaining potential tables for only two generic members of the IID observation class at any given time. Before an observation is made, both of these tables embody the current status of knowledge for the parent variables and a state of ignorance about the values of the IID observations. When an observation is made on a member of the class, an inference engine (such as that of ERGO; see Mislevy, in press, for a simnple worked-through example) is used to absorb the new evidence into the rest of the network, including the potential table for second, not-yet-observed, generic member of the same IID class. The potential table for the now-observed generic member is replaced with a copy of the updated table for the not-yet-observed generic member. The

resulting tables are (1) identical in structure, (2) in agreement as to the marginalized potential for the parent variable(s), and (3) in a state of ignorance about the members of the IID variables, and thereby ready to repeat the process for the next observation.

To specify this sequence more formally, let $f_t(Y,X_j$=unknown) represent the potential table at Time $t$ for the $j^{th}$ not-yet-observed, generic member $X_j$ of the IID class and the (possibly multidimensional) parent variable(s) $Y$. Also instantiated in the network at Time $t$ is the identical potential table for the $j$+1st not-yet-observed member of the IID class, or $f_t(Y,X_{j+1}$=unknown). Suppose the value of $X_j$ , say $x_j$, is now ascertained. The steps required to update beliefs then effect the reduced computing representation of the network are as follows:

1. Update the potential table containing $X_j$, to obtain $f_{t+1}(Y,X_j=x_j)$.

2. Marginalize $f_{t+1}(Y,X_j=x_j)$ with respect to $Y$ to update the clique intersection table, and propagate evidence throughout the rest of the network. This includes obtaining $f_{t+1}(Y,X_{j+1}$=unknown), which now incorporates a marginalized $Y$ distribution that captures the implication of $X_j=x_j$ for expectations about potential values of $X_{j+1}$.

3. Replace $f_{t+1}(Y,X_j=x_j)$ with another copy of the table $f_{t+1}(Y,X_{j+1}$=unknown). This is now thought of as $f_{t+1}(Y,X_{j+2}$=unknown).

Thus, not all not-yet-observed members of the IID class need to be represented explicitly because we know from the structure of the problem that their potential tables are identical to the representative that is contained in the network. That is, for all $n$>1,

$$f_{t+1}(Y,X_{j+n}\text{=unknown}) = f_{t+1}(Y,X_{j+1}\text{=unknown}).$$

Moreover, the potential tables for cliques corresponding to previously-observed values of $X$ have the same marginalization of $Y$. Unless retraction of the information about specific values of previously-observed $X$s is contemplated, it is no longer necessary to carry these potential tables along in future manipulations of the network.

## Example

This is an example from educational testing, with one variable, $\theta$, representing student proficiency and 5 variables, Itm1, ..., Itm5 representing 0/1 (wrong/right) responses to conditionally independent test items. Figure 1 is a directed graph

representation of the problem. The three possible values of $\theta$ are (-1, 0, +1), with prior probabilities of .25, .50, and .25 respectively. The conditional probabilities of a correct response to any item are .10, .40, and .70 respectively. Figure 2 is a join-tree representation of the network, showing five cliques of the form $\{\theta, \text{Itm}_j\}$ connected to the same clique intersection set $\{\theta\}$. There are thus 5 clique potential tables and one clique intersection table. An ERGO run-time file is presented in the Appendix.

[Figures 1 & 2 about here]

Table 1 shows the updating of potential tables as the response sequence (1,1,0,0,1) is absorbed one item at a time. Shaded areas represent potential tables for items already observed. They differ as to the value of the observed response, but agree as to the marginal distribution of their common parent. Note that at any given point in time, the potential tables for all items not-yet-observed are identical. Table 2 shows the same updating, effected with the "update & replace" strategy with only two potential tables for items at any given point in time. For example, after a response of 1 to Item 1 has been observed, the Item 1 potential table collapses to the $x_1$=1 column and the potential table for Item 2 reflects the impact on belief about $\theta$; the Item 1 table is then discarded, and the Item 2 table is duplicated to produce the Item 3 table.

[Tables 1 & 2 about here]

## References

Andersen, S.K., Jensen, F.V., Olesen, K.G., & Jensen, F. (1989). *HUGIN: A shell for building Bayesian belief universes for expert systems* [computer program]. Aalborg, Denmark: HUGIN Expert Ltd.

Andreassen, S., Jensen, F.V., & Olesen, K.G. (1990). Medical expert systems based on causal probabilistic networks. Aalborg, Denmark: Institute of Electronic Systems, Aalborg University.

Gitomer, D.H., Steinberg, L.S., & Mislevy, R.J. (in press). Diagnostic assessment of trouble-shooting skill in an intelligent tutoring system. In P. Nichols, S. Chipman, & R. Brennan (Eds.), *Cognitively diagnostic assessment.* Hillsdale, NJ: Erlbaum.

Lauritzen, S.L., & Spiegelhalter, D.J. (1988). Local computations with probabilities on graphical structures and their application to expert systems (with discussion). *Journal of the Royal Statistical Society, Series B, 50,* 157-224.

Mislevy, R.J. (in press). Probability-based inference in cognitive diagnosis. In P. Nichols, S. Chipman, & R. Brennan (Eds.), *Cognitively diagnostic assessment.* Hillsdale, NJ: Erlbaum.

Noetic Systems, Inc. (1991). ERGO [computer program]. Baltimore, MD: Author.

Pearl, J. (1988). *Probabilistic reasoning in intelligent systems: Networks of plausible inference.* San Mateo, CA: Kaufmann.

Shafer, G., & Shenoy, P. (1988). Bayesian and belief-function propagation. *Working Paper 121.* Lawrence, KS: School of Business, University of Kansas.

## Appendix

An ERGO runtime file for the sample problem is shown below. "q" stands for "$\theta$."

85227296 6 5 2 6 5 1 3 2 4 3 5 4 6 2 1 2 2 2 2 2 3 0 4 0 0 2 6 2 3 4 5 1 2 2.500000e-02
2.250000e-01 2.000000e-01 3.000000e-01 1.750000e-01 7.500000e-02 1 0 1 3 2 6 1 1 3
1.000000e-01 9.000000e-01 4.000000e-01 6.000000e-01 7.000000e-01 3.000000e-01 1
0 1 3 2 6 1 1 4 1.000000e-01 9.000000e-01 4.000000e-01 6.000000e-01 7.000000e-01
3.000000e-01 1 0 1 3 2 6 1 1 5 1.000000e-01 9.000000e-01 4.000000e-01 6.000000e-01
7.000000e-01 3.000000e-01 1 0 1 3 2 6 1 1 6 1.000000e-01 9.000000e-01 4.000000e-01
6.000000e-01 7.000000e-01 3.000000e-01 Itm1 ITM2 ITM3 ITM4 ITM5 q 1 0 1 0 1 0 1
0 1 0 -1 0 1

**Table 1**

**Trace of Potential Tables and Clique Intersection Table for Five Responses to IID Responses**

**INITIAL STATUS**

| θ | |
|---|---|
| θ=-1 | .250 |
| θ=0 | .500 |
| θ=+1 | .250 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

**AFTER ITEM 1 = 1**

| θ | |
|---|---|
| θ=-1 | .063 |
| θ=0 | .500 |
| θ=+1 | .438 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .063 | 0 |
| θ=0 | .500 | 0 |
| θ=+1 | .438 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

**AFTER (1,1)**

| θ | |
|---|---|
| θ=-1 | .012 |
| θ=0 | .390 |
| θ=+1 | .598 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .012 | 0 |
| θ=0 | .390 | 0 |
| θ=+1 | .598 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .012 | 0 |
| θ=0 | .390 | 0 |
| θ=+1 | .598 | 0 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

**AFTER (1,1,0)**

| θ | |
|---|---|
| θ=-1 | .026 |
| θ=0 | .552 |
| θ=+1 | .422 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .026 | 0 |
| θ=0 | .552 | 0 |
| θ=+1 | .422 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .026 | 0 |
| θ=0 | .552 | 0 |
| θ=+1 | .422 | 0 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | 0 | .026 |
| θ=0 | 0 | .552 |
| θ=+1 | 0 | .422 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .003 | .023 |
| θ=0 | .221 | .331 |
| θ=+1 | .296 | .127 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .003 | .023 |
| θ=0 | .221 | .331 |
| θ=+1 | .296 | .127 |

**AFTER (1,1,0,0)**

| θ | |
|---|---|
| θ=-1 | .048 |
| θ=0 | .688 |
| θ=+1 | .263 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .048 | 0 |
| θ=0 | .688 | 0 |
| θ=+1 | .263 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .048 | 0 |
| θ=0 | .688 | 0 |
| θ=+1 | .263 | 0 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | 0 | .048 |
| θ=0 | 0 | .688 |
| θ=+1 | 0 | .263 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | 0 | .048 |
| θ=0 | 0 | .688 |
| θ=+1 | 0 | .263 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .005 | .044 |
| θ=0 | .275 | .413 |
| θ=+1 | .184 | .079 |

**AFTER (1,1,0,0,1)**

| θ | |
|---|---|
| θ=-1 | .010 |
| θ=0 | .592 |
| θ=+1 | .397 |

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .010 | 0 |
| θ=0 | .592 | 0 |
| θ=+1 | .397 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .010 | 0 |
| θ=0 | .592 | 0 |
| θ=+1 | .397 | 0 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | 0 | .010 |
| θ=0 | 0 | .592 |
| θ=+1 | 0 | .397 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | 0 | .010 |
| θ=0 | 0 | .592 |
| θ=+1 | 0 | .397 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .010 | 0 |
| θ=0 | .592 | 0 |
| θ=+1 | .397 | 0 |

Table 2

Reduced Representation Trace of Potential Tables and Clique Intersection Table

**INITIAL STATUS**

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .025 | .225 |
| θ=0 | .200 | .300 |
| θ=+1 | .175 | .075 |

| | θ |
|---|---|
| θ=-1 | .250 |
| θ=0 | .500 |
| θ=+1 | .250 |

**AFTER ITEM 1 = 1**

| | Itm1=1 | Itm1=0 |
|---|---|---|
| θ=-1 | .063 | 0 |
| θ=0 | .500 | 0 |
| θ=+1 | .438 | 0 |

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

| | θ |
|---|---|
| θ=-1 | .063 |
| θ=0 | .500 |
| θ=+1 | .438 |

**BEFORE ITEM 2**

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

| | θ |
|---|---|
| θ=-1 | .063 |
| θ=0 | .500 |
| θ=+1 | .438 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .006 | .056 |
| θ=0 | .200 | .300 |
| θ=+1 | .306 | .131 |

**AFTER (1,1)**

| | Itm2=1 | Itm2=0 |
|---|---|---|
| θ=-1 | .012 | 0 |
| θ=0 | .390 | 0 |
| θ=+1 | .598 | 0 |

| | θ |
|---|---|
| θ=-1 | .012 |
| θ=0 | .390 |
| θ=+1 | .598 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

**BEFORE ITEM 3**

| | θ |
|---|---|
| θ=-1 | .012 |
| θ=0 | .390 |
| θ=+1 | .598 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .001 | .011 |
| θ=0 | .156 | .234 |
| θ=+1 | .418 | .179 |

**AFTER (1,1,0)**

| | θ |
|---|---|
| θ=-1 | .026 |
| θ=0 | .552 |
| θ=+1 | .422 |

| | Itm3=1 | Itm3=0 |
|---|---|---|
| θ=-1 | 0 | .026 |
| θ=0 | 0 | .552 |
| θ=+1 | 0 | .422 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .003 | .023 |
| θ=0 | .221 | .331 |
| θ=+1 | .296 | .127 |

**BEFORE ITEM 4**

| | θ |
|---|---|
| θ=-1 | .026 |
| θ=0 | .552 |
| θ=+1 | .422 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | .003 | .023 |
| θ=0 | .221 | .331 |
| θ=+1 | .296 | .127 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .003 | .023 |
| θ=0 | .221 | .331 |
| θ=+1 | .296 | .127 |

**AFTER (1,1,0,0)**

| | θ |
|---|---|
| θ=-1 | .048 |
| θ=0 | .688 |
| θ=+1 | .263 |

| | Itm4=1 | Itm4=0 |
|---|---|---|
| θ=-1 | 0 | .048 |
| θ=0 | 0 | .688 |
| θ=+1 | 0 | .263 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .005 | .044 |
| θ=0 | .275 | .413 |
| θ=+1 | .184 | .079 |

**AFTER (1,1,0,0,1)**

| | θ |
|---|---|
| θ=-1 | .010 |
| θ=0 | .592 |
| θ=+1 | .397 |

| | Itm5=1 | Itm5=0 |
|---|---|---|
| θ=-1 | .010 | 0 |
| θ=0 | .592 | 0 |
| θ=+1 | .397 | 0 |

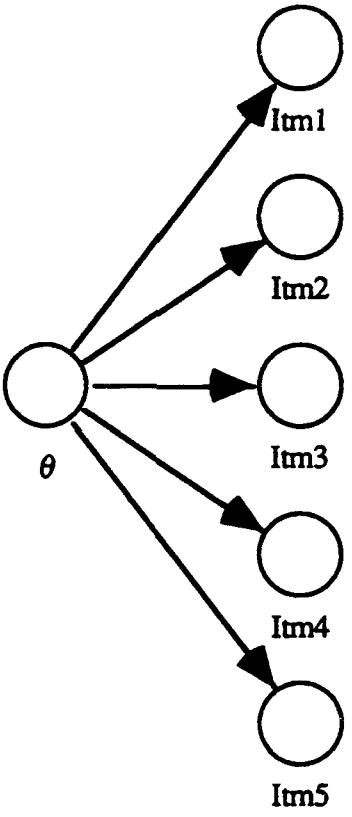


Figure 1

Directed Graph for Example

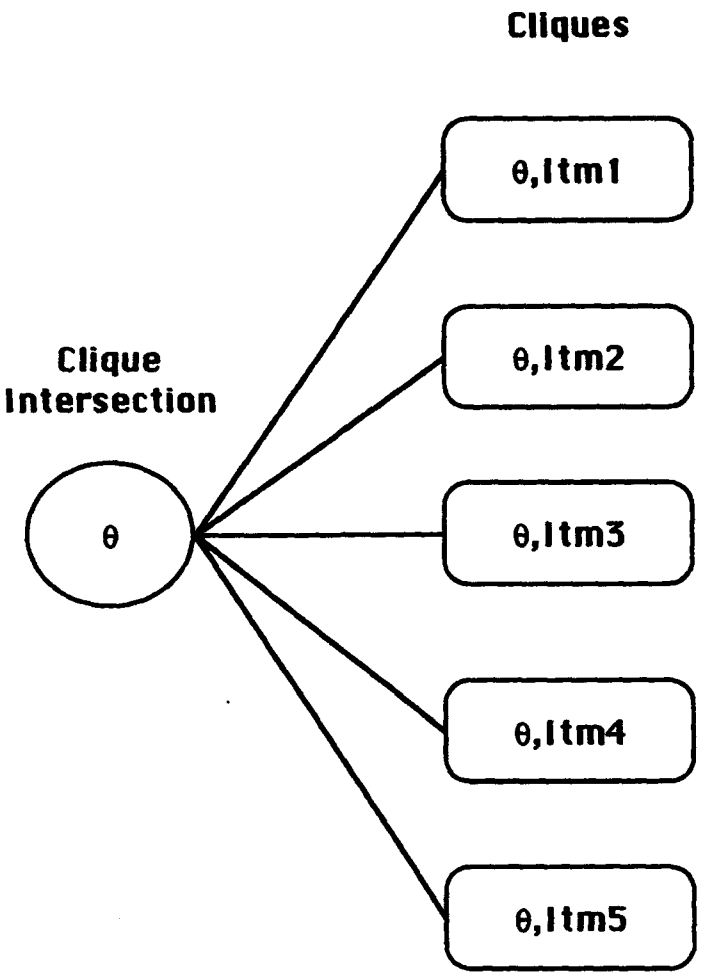


Figure 2

Join Tree for Example

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